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ESTIMATE OF THE TEMPERATURE FIELD IN A COMPRESSIBLE MEDIUM

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Short-duration dynamic loading causes the development and propagation of shock waves (SW) in a continuous medium. The intensity of the shock waves depends on the intensity of the load applied and the physicomechanical characteristics of the material. As a result of high compression, the material is heated [1]. As the SW progresses, the region of material compression shifts, and the temperature of the medium changes. Estimates of the temperature field in a compressible medium and the residual heating of the material are of great interest for practical applications.

The methods for calculating the elastic potential and the temperature at the wave front are presented in [2, 3]. The Hugoniot adiabats, the atomic interaction potentials, and the Grüneisen coefficients have been determined in [4] for a number of metals. A detailed survey of the literature is provided in [5] along with an analysis of the known equations of state (see also [2, 4]). The methods of estimating the temperature on the Hugoniot adiabat and the residual temperature are given in [6-9].

The state of the continuous medium is described by the following system of equations [10]. The equation of motion is

$$\frac{\rho_0}{V}\frac{\partial u}{\partial \tau} = -\frac{\partial}{\partial r}(p - S_r) + \frac{(k-1)(S_r - S_\theta)}{r}, \tag{1}$$

where $V = \rho_0/\rho_0$; ρ_0 and ρ are the initial and the present values of the density, respectively, u is the mass velocity, r is the present radius, p is the mean stress, and S_r and S_θ are the components of the stress deviator; k = 1, 2, and 3 for the two-dimensional, cylindrical, and spherical cases, respectively.

The continuity equation is given by

$$\frac{\dot V}{V} = \frac{1}{r}\frac{\partial (r^{k-1}u)}{\partial r}. \tag{2}$$

with an allowance for the thermal conductivity, the energy equation is given by

$$\dot E = p\dot V - V\left(S_r\dot\varepsilon_r + (k-1)S_\theta\dot\varepsilon_\theta\right) + \frac{k}{r^{k-1}}\frac{\partial}{\partial r}\left(r^{k-1}\varkappa\frac{\partial T}{\partial r}\right). \tag{3}$$

Here, κ is the thermal conductivity coefficient, T is the temperature, and E is the total energy. The expressions for the strain are the following:

$$\varepsilon_r = \frac{\partial e}{\partial r} - \frac{1}{2}\left(\frac{\partial e}{\partial r}\right)^2, \quad \varepsilon_\theta = \frac{e}{r} - \frac{1}{2}\left(\frac{e}{r}\right)^2 \tag{4}$$

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(e is the displacement). The relationship between the stress and strain deviators is provided by Hooke's law.

The Mie–Grüneisen equation is contemplated as the equation of state:

$$p = p_{el} + \gamma_0(E - E_{el}) \tag{5}$$

($p_{e\ell}$ and $E_{e\ell}$ are the elastic pressure and energy, respectively, and γ_0 is the Grüneisen coefficient). The following was obtained for $p_{e\ell}$ in [9]:

$$p_{el} = p_0 \exp(\gamma_0\eta) + p_H - \gamma_0 R(\eta) - G(\eta), \tag{6}$$

where $\eta = 1 - \rho_0/\rho$, and p_H is the pressure on the shock adiabat; the $R(\eta)$ and $G(\eta)$ functions are given by

$$R(\eta) = \rho\alpha^2\left(\frac{\eta^2}{2(1-\beta\eta)^2} - \frac{\eta}{\beta(1-\beta\eta)} - \frac{\ln(1-\beta\eta)}{\beta^2}\right); \tag{7}$$

$$G(\eta) = \gamma_0 \exp(\gamma_0\eta)\int_0^{\eta} \exp(-\gamma_0\eta) R(\eta)\, d\eta \tag{8}$$

(α and β are the coefficients of the well-known expression $D = \alpha + \beta u$, which relates the velocity of the SW front D to the mass velocity of the substance u). According to [9], we write (5) as follows: $p = p_0 \exp(\gamma_0\eta) + \Pi + \gamma_0\rho_0 c_V T$ [c_V is the specific heat, and $\Pi = p_H - \gamma_0 R(\eta) - G(\eta)$]. Under normal conditions, $p = 0$ and $\eta = 0$ (we neglect the atmospheric pressure), so that

$$p_0 = -\rho_0\gamma_0 c_V T; \tag{9}$$

$$E_y = -c_V\rho_0 T. \tag{10}$$

Utilizing (9) and (10), we pass to the equation of state where all the parameters are measured from zero:

$$p = p_0 \exp(\gamma_0\eta) + \Pi + \gamma_0(E - E_{el}) = p_{el} - p_0 + \rho_0\gamma_0 c_V t \tag{11}$$

(E, $E_{e\ell}$, and t are the total intrinsic energy, the elastic energy, and the temperature, respectively).

The increment of the thermal part of the intrinsic energy E_T is determined by the difference $dE_T = dE - dE_{e\ell}$. The temperature increment is calculated as follows:

$$dT = (dE - dE_{el})/\rho_0 c_V. \tag{12}$$

Equation (12) accounts for the process of linkage between the strain and temperature fields. Actually, assuming that there is only elastic strain at a point, the total energy increment is $dE = S_{ij}d\varepsilon_{ij} - pdV$, while the elastic energy increment is given by

$$dE_{el} = S_{ij}d\varepsilon_{ij} - p_{el}dV. \tag{13}$$

With an allowance for (9) and (10), it is evident from (13) that the following expression holds for the temperature increment:

$$dt \sim -(T_0 + t)dV, \tag{14}$$

which characterizes the temperature change due to the linkage between the strain and heating processes. The above relationships generalize the consideration of linkage in [11] to encompass the case where $t \sim T_0$, since the assumption that $t \ll T_0$ is no longer used. Consideration of the linkage in the classical formulation results in the fact that the temperature of the body remains the same after the motion is damped. According to [11], the derivative of the temperature with respect to time, obtained from the energy equation, is given by

$$\dot{T} = \chi\Delta T - \frac{3\lambda + 2\mu}{\rho_0 c_V} 3\alpha_T T_0 \dot{\varepsilon}_{V0}, \tag{15}$$

where λ and μ are the Lamé coefficients, α_T is the thermal expansion coefficient, ε_{V0} is the volume strain, and $\chi = \kappa/\rho_0 c_V$. After integrating (15) with respect to time from $\tau = 0$ to $\tau \to \infty$ and over the volume v, we obtain

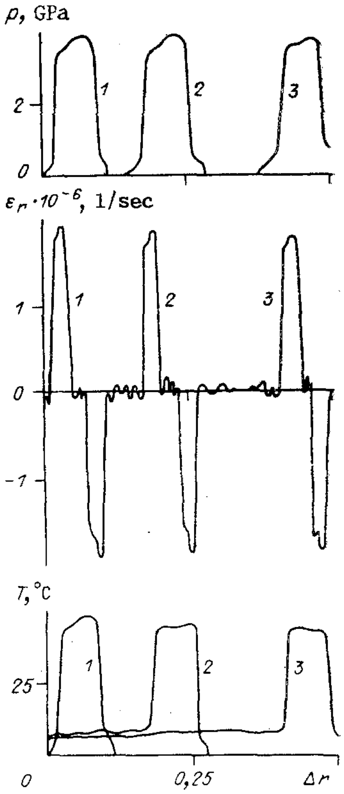

Fig. 1

$$T|_{\tau\to\infty} - T|_{\tau=0} = \chi \int\int \frac{\partial T}{\partial n} dS - \frac{3\lambda + 2\mu}{\rho_0 c_V} 3\alpha_T T \varepsilon_{VO}.$$

In the absence of heat exchange through the boundary, and considering that $\lim_{\tau\to\infty} \varepsilon_{VO} = 0$, we find that the temperature increment is equal to zero.

Consideration of the linkage in accordance with (14) does not allow us to integrate the energy equation in the above manner, since the coefficient in front of $\dot{\varepsilon}_{VO}$ depends on the temperature itself, and, therefore, the temperature increment is a positive quantity in this case. Actually, by dividing the energy equation by T, we obtain according to (14),

$$\frac{d \ln T}{d\tau} = \frac{\chi}{T} \Delta T - \frac{3\lambda + 2\mu}{\rho_0 c_V} 3\alpha_T \dot{\varepsilon}_{VO} = \nabla\left(\frac{\chi}{T} \nabla T\right) + \frac{\chi}{T^2} (\nabla T)^2 - \frac{3\lambda + 2\mu}{\rho_0 c_V} 3\alpha_T \dot{\varepsilon}_{VO}$$

Performing integration over the volume and with respect to time, we write

$$T|_{\tau\to\infty} - T|_{\tau=0} \simeq \int\int \frac{\chi}{T} \frac{\partial T}{\partial n} dS\, d\tau + \int\int \frac{\chi}{T^2} (\nabla T)^2 dv\, d\tau - \frac{3\lambda + 2\mu}{\rho_0 c_V} 3\alpha_T \varepsilon_{VO},$$

whence

$$T|_{\tau\to\infty} - T|_{\tau=0} \simeq \int\int \frac{\chi}{T^2} (\nabla T)^2 dv\, d\tau > 0.$$

As a calculation example, we consider an aluminum plate with a thickness of 0.5 cm, the surface of which is acted upon by a rectangular pressure pulse with an amplitude of 4 GPa and a duration of 0.15 μsec, while the other plate surface is stress-free. The initial plate temperature is equal to zero. The system of equations (1)-(8) and (12) was used for determining the change in the stressed-strained state and the temperature field. The problem was solved in the linkage formulation by means of the method of finite differences. The scheme of second-order accuracy and straight-through calculation was used, and artificial viscosity of the Neumann-Richtmayer type was introduced.

Figure 1 shows the distributions in the direction of the plate thickness of the mean stress profile, the radial strain rate, and the temperature field at different instants of time (curves 1-3 correspond to τ = 0.16, 0.48, and 0.94 μsec, respectively). It is evident that the pulse progression in the direction of the plate thickness is accompanied by a high strain rate and the development of a temperature field. As the compression region shifts, there is a gradual attenuation of the pressure pulse, caused by dissipative processes, which leads to a reduction in temperature.

The shape of the mechanical pulse acting on the surface and its amplitude were varied during the investigation, and plates made of different materials were used. Calculations show that the temperature field profile is determined by the profile of the load applied. The temperature of the medium in the compression region and the residual temperature increase with the amplitude of the load applied.

Using the relationships given in [10], we can readily extend the pattern described to the plastic flow range. Thus, as a result of impact compression, residual heating also occurs without plastic strain. The solution of the problem stated makes it possible to estimate the temperature field arising under dynamic loading in a medium in the moderate pressure range, which is important with regard to practical applications.

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